where $\alpha(\theta)$ is the spanwise angle-of-attack distribution. For an elliptic wing the chord distribution is $c(\theta)=c_o\sin\theta$ where c_o is the length of the ellipse minor axis, and the unknown constants A_n can then be solved for explicitly for any twist distribution. The sinusoidal twist

$$\alpha(\theta)=\sum_{r=1}^{\infty}\alpha_r\sin r\theta+\sum_{r=1}^{\infty}\beta_r\cos r\theta \tag{3}$$

in particular, gives the solution

$$\left(n+\frac{4s}{\pi c_o}\right)A_n=-\frac{16n}{\pi}\sum_{r=1}^{\infty}{}' r\alpha_r/[(r+n)^2-1][(r-n)^2-1] + \sum_{r=1}^{\infty}\beta_r d_r \tag{4}$$

where

$$\begin{aligned} d_r &= 1 ;\ \text{if}\ (r-n)=1 \\ &= -1 ;\ \text{if}\ (n-r)=1 \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{5}$$

and Σ' denotes summation only over even $(n+r)$. This result is simpler than those of Ref. 1 which contains Bessel functions of the first kind and Chebyshev polynomials of the second kind. The flat plate result is well known and may be added to the solution.

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Comment on Papers by B. W. Roberts and K. R. Reddy

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PUBLISHED simultaneously by members of the same university department, these two papers are obviously closely related, and it seems permissible to submit one comment on those aspects which are generally common to both.

Roberts' specific criticisms of filling time concepts† are certainly well taken. The fact that they were put forward in the first place is a quirk of history, in that the first parachutes were developed empirically, and to a certain extent, a philosophy of "cut and try" has pervaded the field ever since. There has been little motivation to put scarce R&D money into theoretical studies because: a) existing theoretical predictions were hopelessly far removed from experimental observation[6]; b) "cut and try" is not prohibitively expensive; and c) although not complex by comparison with many aerodynamic problems which we have solved satisfactorily in the past, the opening process cannot be solved by a single "Ph.D. thesis" level of effort. A properly planned, multifaceted attack on the problem is needed, for which, as we have suggested, there is little official motivation.

Index categories: Aircraft Deceleration Systems; Nonsteady Aerodynamics.

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†References 3-5 are examples he cited; more are cited in Ref. 7.

‡Reference 7 was submitted in November 1970, published February 1973 with the legend "Received June 1972."

The papers of Roberts[1] and Reddy[2] are to be welcomed as serious preliminaries on one aspect of the inflation process, but the reader might be tempted to read more into these papers than the authors perhaps intended. Apart from the fact that the three-dimensional problem is treated as two-dimensional, a number of important terms are omitted from the analysis. Most of these were at least outlined in Ref. 7,‡ a paper which has apparently escaped the attention of both authors. The most important considerations omitted would seem to be as follows:

1) When the canopy is opening, it moves toward the payload. In the simplest case, treated by Reddy, the distance between the store and the wedge apex is, in Reddy's notation

$$X=b\cos\alpha+L\cos\delta=b\cos\alpha+[1-(b/L)^2\sin^2\alpha]^{1/2} \tag{1}$$

If V_s is the store's velocity along the flight path, and if the store mass greatly exceeds the virtual canopy mass the velocity of the canopy is

$$V_c=V_s+b\sin\alpha\frac{d\alpha}{dt}\left[1+\frac{1}{[1-(b/L)^2\sin^2\alpha]^{1/2}}\right] \tag{2}$$

Note that as $\alpha\to 0$ and/or $b/L\to 0$

$$V_c\to V_s+b\sin\alpha(d\alpha/dt) \tag{2a}$$

With the same assumptions, the canopy acceleration is

$$\frac{dV_c}{dt}=\frac{dV_s}{dt}+b\left[1+\frac{1}{[1-(b/L)^2\sin^2\alpha]^{1/2}}\right]\left[\left(\frac{d\alpha}{dt}\right)^2\cos\alpha+\frac{d^2\alpha}{dt^2}\sin\alpha\right]+\frac{b\sin^2\alpha(b/L)^2(d\alpha/dt)^2}{[1-(b/L)^2\sin^2\alpha]^{3/2}} \tag{3}$$

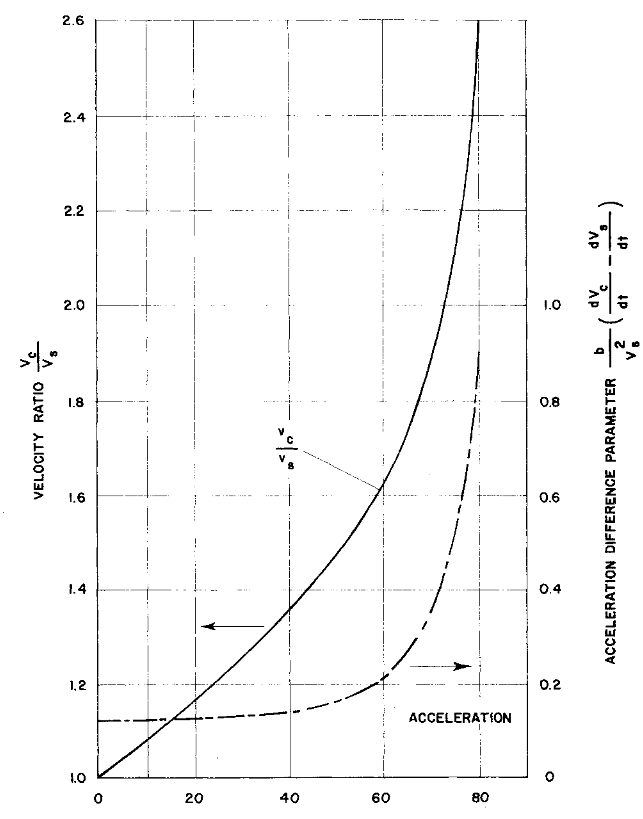

Fig. 1 Canopy velocity and acceleration parameters for infinite payload mass, inextensible shrouds, constant angular inflation velocity and $\alpha_f=70°$ $(U_0t_f/b)=5.0$, $b/L=1.0$.

To give some physical "feel," these two equations are plotted in Fig. 1 for a representative case, on the assumption that the wedge is opening at a constant velocity α, that the filling time is $5b/u_o$ and that the fully open angle is $\alpha_f = 70°$. Note that the canopy velocity can approach twice the store velocity, and that the canopy acceleration dV_c/dt can be of the order of 100 g greater than that of the store. The latter observation explains the (as was previously thought) extraordinarily high static pressures ($>5q$) measured by Melzig[9] under infinite mass conditions. It can also be seen that this relative motion effect alone (originally pointed out by Foote and Giever[8]) gives rise to overfill and a subsequent damped oscillation, because of the $\ddot{\alpha}$ and $\dot{\alpha}$ terms in conjunction with the canopy's virtual mass. A theory which does not include this effect cannot be expected to agree with experiment.

2) In a practical parachute, the risers, shrouds, and canopy have a finite resiliency which is important to the filling dynamics. Reducing the stiffness increases opening time, according to the writer's theory,[7] and also according to some experimental observations by Heinrich,[10] below a given stiffness (at a given speed) the parachute will not open at all. One might say that shroud stiffness dominates the filling dynamics. The authors do not consider this parameter. For illustrative purposes, take the V-gutter model of Reddy, and assume that the shrouds have a total stiffness K, and stretch an amount $\mathcal{L}$. Then

$$L = L_o + \mathcal{L}; F/\cos\delta = K\mathcal{L}; L = L_o + F/K \cos\delta \tag{4}$$

Then, from Eq. (1), the distance between the store and the canopy apex is

$$X = b\cos\alpha + L_o\cos\delta + (F/K) \tag{5}$$

$$V_c = V_s + b\sin\alpha(d\alpha/dt) + L_o\sin\delta(d\delta/dt) + (1/K)(dF/dt) \tag{6}$$

$$(dV_c/dt) = (dV_s/dt) + b\cos\alpha[(d\alpha/dt)]^2 + b\sin\alpha(d^2\alpha/dt^2) + L_o\cos\delta[(d\delta/dt]^2 + L_o\sin\delta(d^2\delta/dt^2) + (1/K)(d^2F/dt^2) \tag{7}$$

Where α and δ are related by

$$\sin\delta = (b/L)\sin\alpha = b\sin\alpha/[L_o + (F/K\cos\delta]$$

i.e.,

$$\sin\alpha = (L_o/b)\sin\delta + (F/Kb)\tan\delta \tag{8}$$

Notice that the canopy force and its derivatives are closely coupled in a complex way with V_c, V_s, and their first derivatives, the terms which to a large extent govern the canopy force.

3) Reference 7 shows that the line stretch at the commencement of opening has a major influence on opening time and loads, and that this probably explains, at least in part, the great variability in experimental observations.

4) The authors do not consider the vent flow.

5) The foregoing might be regarded as omissions. It would also have been interesting to see a comparison between the forces computed by the authors and the equivalent "added" or "virtual mass" results. This approach first suggested for the parachute problem by von Karman,[11] has been of considerable utility in many other problems, has the merit of simplicity, and can be readily applied to the real axi-symmetric problem. Examples of prior use are Munk's[12] analysis of bodies of revolution, R. T. Jones'[13] analysis of slender wings, and the calculation of the force and moment derivatives on a planning plate.[14] The writer cannot immediately put his hand to the added mass values for V-shaped or parabolic gutters, but the point may be illustrated simply by taking the elliptic cylinder result to apply to the V-gutter case, the semi-width normal to the flow being $b\sin\alpha$. Then the added mass per unit span is roughly

$$m' = \rho\pi b^2\sin^2\alpha$$

If V_c is the canopy velocity along the flight path, the inertial force per unit length is

$$\text{Force} = (d/dt)(m'V_c) = \rho\pi b^2 [(dV_c/dt)\sin^2\alpha + 2V_c(d\alpha/dt)\sin\alpha] \tag{9}$$

If we substitute Eq. (6) and (7) in Eq. (9), we obtain, per unit length:

$$(\text{Force}/\rho\pi b^2) = (dV_s/dt)\sin^2\alpha + 2V_s(d\alpha/dt)\sin\alpha (\text{von Karman's result})$$

The additional inertial and line stretch terms

$$+ b\sin^2\alpha[\cos\alpha(\frac{d\alpha}{dt})^2 + \sin\alpha\frac{d^2\alpha}{dt^2} + \frac{L_o}{b}\cos\delta(\frac{d\delta}{dt})^2 + \frac{L_o}{b}\sin\delta\frac{d^2\delta}{dt^2} + \frac{1}{bK}\frac{d^2F}{dt^2}] + 2b\frac{d\alpha}{dt}\sin\alpha[\sin\alpha(\frac{d\alpha}{dt}) + \frac{L_o}{b}\sin\delta\frac{d\delta}{dt} + \frac{1}{bK}\frac{dF}{dt}] \tag{10}$$

The "quasi-static" forces due to ram pressure must be added to this result, or course, as indicated in Ref. 7. An additional equation is required to determine the variation of α with time, and hence $\dot{\alpha}$ and $\ddot{\alpha}$, and this can be obtained from the workof Roberts or Reddy, or by employing the inflation concepts of Ref. 7. Because of the important effects of shroud tension in inhibiting inflation, the α equation will be closely coupled with Eq. (10).

Roberts concludes his paper with the claim that "the current theory, it is believed, is more extensive and more rigorous in application than any other currently available." The writer was careful to avoid such claims in Ref. 7 (although he does share Roberts' views on the inadequacy of the earlier literature) because he regarded his analysis as a broad-brush sketch of a fairly complex dynamically coupled fluid dynamic problem. But he submits that, because of Roberts' and Reddy's preoccupation with ultra-simplified (but solvable) models of the canopy flow, the analysis of Ref. 7 gives a better insight into the problem, since it addresses important terms neglected in their analysis. From either base, we have a long way to go before acceptable agreement with experiment can be achieved, so that parachutes can be designed and optimized on the computer.

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Reply by Authors to P.R. Payne

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WE HAVE read Payne's comments with interest, and it is agreed that the papers in question are initial steps towards understanding the aerodynamics of some idealized, inflatable structures. Roberts considered an inflating, two-dimensional parabolic shell, while Reddy examined a two-dimensional, wedge-shaped shell.

We chose these types of structures as only one parameter, namely h or α, is necessary to completely define the structural shape at any instant of time. In other words, the structure is in its most elementary form and exists as a single degree-of-freedom system. If one was to consider an inelastic, membrane structure we would need an infinite number of degrees of freedom to completely define the structure's shape.

Therefore, we wished to idealize the structure in this manner so that the way was clear for a detailed consideration of the unsteady flow about such bodies. It is not intended that the structural or the aerodynamic argument be exactly applicable to the parachute, but it is not unreasonable to see that Reddy's structure is physically realizable as a hinged flat plate. This hinged plate is capable of aerodynamic inflation. The parabola, on the other hand, is only a mathematical artifice.

Now both these elementary structures can be mapped using well known conformal transformations. From these functions is is possible to calculate the pressure distribution on the structure as a function of α, $\dot{\alpha}$, $\ddot{\alpha}$, β, and $\dot{\beta}$ in Reddy's notation. Thus we have the pressure distribution on the structure as a function of two variables α and β. Reddy has then considered in his concluding example that the freestream velocity approaching the wedge is constant, that is $\beta=1$, $\dot{\beta}=0$, and this assumption is only an approximation to the infinite mass case. With this assumed form for β, Reddy only needed *one* structural equation to solve for $\alpha(t)$, and this appears as Eq. (14) in his paper. It must be stressed that this is a moment equation applied about the hinge. In this manner we have simultaneously satisfied the *structural and aerodynamic equations* without any recourse to a filling time notion.

The whole thrust of our argument does not depend on any filling time postulate given in some prior literature. Indeed, this same filling time notion is also *assumed* by Payne in Eqs. (2) and (3) of his Ref. 7. We would claim that such an assumption should not be necessary nor is it reasonable. We should also stress that Reddy's Eq. (15) is a *moment equation* which is only loosely coupled to the virtual mass, etc., of the canopy in *translational motion*. For instance, c_1 in Reddy's Eq. (15) is the virtual moment of inertia of the system, c_2 is the damping moment derivative, and so on. These latter quantities relate to *rotation about the hinge not translation of the wedge*. We feel that this aspect may not have been emphasized enough in our work, and we regret any possible confusion.

We make it quite clear that we accept Payne's criticisms relating to our neglect of the line stiffness, canopy elasticity, etc. Furthermore, the virtual mass referred to in Sec. V of Payne's comment is in fact buried in $\int pds$ of Reddy's Eq. (14), and the virtual mass can be found from the appropriate terms in Reddy's integral or in Wang and Wu's paper, "Small Time Behaviour of Unsteady Cavity Flows." [1] In conclusion, we agree with Payne's last paragraph and add that even a simple, hinged flat plate is a structure which is dynamically coupled with the fluid flow in a rather complex matter.

Finally it is our intention to present at the next AIAA Aerodynamic Deceleration Systems Conference, Nov. 17-19, 1975, an analysis of an n degree-of-freedom hinged system. This latter approach we believe is not an "ultra - simplified model of the canopy flow," nor an unreasonable structural model.

Received December 9, 1974.
Index categories: Aircraft Deceleration Systems; Nonsteady Aerodynamics.
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